HEI-003-1161003

HEI-003-1161003 **Seat No.** __________

M. Sc. (Sem. I) (CBCS) Examination

November / December – 2017

Mathematics : 1003

(Topology-I) (New Course)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries two marks) **14**

(1) If every subset of X is closed set of X then the topology on X is ......topology.

(2) In a topological space X _________ and _________ are both open and closed set.

(3) In $\mathbb{R}$ the closure of the set of rational numbers is _________.

(4) If A contains all its limit points then A is _________ set.

(5) The set of natural numbers is a closed set in $\mathbb{R}$ when $\mathbb{R}$ has _________ topology.

(6) The number of components of a disconnected space is at least _________.

(7) A subset G of X is open if and only if G° = _________.

2 Attempt any two :

(a) Give an example to show that denumerable intersection of open set need not be open. **7**

(b) Let A be a subset of X. Prove that $X \setminus \overline{A} = (X \setminus A)^0$ **7**

(c) Let A subset of X and B subset of Y. Prove that : **7**

(1) $\overline{(A \times B)} = \overline{A} \times \overline{B}$.

(2) Prove that for any subset A of X $(A^0)^0 = A^0$.

3 All are compulsory :

(a) Give the definition of separation of a space X. Find one separation for the subspace of natural numbers and deduce that this space is disconnected. **6**

(b) Prove that every component is a maximal connected set and it is a closed set. **4**

(c) Prove that every path connected space is connected. **4**

OR

3 All are compulsory :

(a) Prove the subspace (0, 1) is homeomorphic to (a, b) of $\mathbb{R}$. **5**

(b) Suppose f : X $\rightarrow$ Y is continuous and Z is a subspace of X. Then prove that the function f / Z : Z to Y is continuous. **4**

(c) Prove that the set of all natural numbers has no limit point in $\mathbb{R}$ when $\mathbb{R}$ has the standard topology. **5**

4 Attempt any two :

(a) Prove that X × Y is a locally connected if and only if X and Y are locally connected. **7**

(b) Give an example of a connected space which is not locally connected and give an example of a locally connected space which is not connected. **7**

(c) Suppose (X, τ) is a topological space where $\tau = \tau$ (d), for some metric d on X, let $E \subset X$ and $x \in X$. Prove that $x \in \overline{E}$ if and only if there is a sequence in E which converges to x. **7**

5 Do as directed (Each question carries two marks) **14**

(1) Give the definition of a convex subset of a simply ordered set.

(2) Give an example of a closed subset of $\mathbb{R}$ with discrete topology which is not a closed subset of $\mathbb{R}$ with standard topology.

(3) Give an example of a subset of $\mathbb{R}$ which is closed when $\mathbb{R}$ has standard topology but it is not closed when $\mathbb{R}$ has co - finite topology.

(4) Find all interior points of the set of all rational numbers when $\mathbb{R}$ has the standard topology.

(5) Give an infinite subset of $\mathbb{R}_l$, which is both open and closed.

(6) Give the definition of the dictionary order on $\mathbb{R} \times \mathbb{R}$.

(7) Give the definitions of closure and the interior of any subset A of a topological space X.